A GRAPHICAL TECHNIQUE FOR DETERMINING EVAPOTRANSPIRATION BY THE THORNTHWAITE METHOD

WAYNE C. PALMER

U. S. Weather Bureau, Washington, D. C.

and

A. VAUGHN HAVENS

Department of Meteorology, Rutgers University, New Brunswick, N. J.

[Manuscript received December 4, 1957; revised March 6, 1958]

ABSTRACT

Generalized diagrams for the determination of evapotranspiration and potential evapotranspiration in accordance with the empirical equations of Thornthwaite are presented.

1. INTRODUCTION

The loss of water from the earth to the atmosphere by transpiration from vegetation and by direct evaporation constitutes an important part of the water balance problem. However, direct measurement of these factors has proved to be extremely difficult, and this inherent difficulty has led to the development of a number of formulas designed to estimate water loss directly from meteorological data.

It is not the purpose of this article to review or evaluate the various methods which have been developed, but it would appear that some mention of the various approaches to the problem is in order. A detailed and excellent review has been published by van Wijk and de Vries [1]; and an enlightening analysis of the problems involved was presented by Penman [2].

The list of references following this article includes some of the principal publications devoted to the problem of estimating soil moisture loss directly from meteorological data and will suffice to indicate the nature of the various approaches used. They fall into three groups: (1) Those involving the flux of water vapor; (2) those utilizing the heat balance of the evaporating or transpiring surface; and (3) those which use an empirically determined relationship between evapotranspiration and one or more of the meteorological factors involved.

None of these methods provides a completely adequate solution to the problems of evaporimetry because none is free from assumptions, arbitrary constants, or technical difficulties of observation and measurement. In spite of the shortcomings, a number of workers have surmised that these methods enable the climatologist to estimate *total* evapotranspiration from a sizable field more accurately than a soil scientist can measure it.

The Thornthwaite method [3] was developed from rainfall and runoff data for several drainage basins. The result is basically an empirical relationship between potential evapotranspiration and mean air temperature. In spite of the inherent simplicity and obvious limitations of the method, it does surprisingly well. It is not necessarily the most accurate method, nor does it have the soundest theoretical basis. On the contrary, these distinctions probably belong to one of the vapor flux or heat balance methods. Among the more obvious shortcomings of Thornthwaite's empirical relationship is the inherent assumption that a high correlation exists between mean temperature and some of the other pertinent parameters such as radiation, atmospheric moisture, and wind. While such limitations may be relatively unimportant under certain circumstances, they are at times of the utmost importance; and Thornthwaite and Mather [4], recognizing that solar radiation and atmospheric turbulence are the important factors in natural evaporation, have stated that the problem of developing a formula for potential evapotranspiration remains unsolved.

Although ease of application is not a suitable criterion of adequacy, it is often a primary consideration for use. The vapor flux and heat balance methods require meteorological data which are either not observed or are observed only at a few widely separated points. On the other hand, Thornthwaite's empirical formula can be used for any location at which daily maximum and minimum temperatures are recorded. It is this simple universal applicability rather than any claim to outstanding accuracy which has led to the widespread use of this method.

It is not the purpose of this paper to make a case either for or against this or any other method. Rather, recognizing the widespread interest in computing and testing results of Thornthwaite's method, it is the purpose to present a graphical solution which will make such computations easier.

2. GRAPHICAL SOLUTION OF THORNTHWAITE METHOD

Thornthwaite's basic formula in the form for computing monthly potential evapotranspiration is

$$e=1.6\ (10T/I)^a$$

where e=monthly potential evapotranspiration (cm.)
T=monthly mean temperature (° C.)
I=a heat index which is a constant for a given location and is the sum of 12 monthly index values i, where i is a function of the monthly normal temperatures. (See table 1.)
a=an empirically determined exponent which is a function of I, $a=6.75\times10^{-7}\ I^3-7.71\times10^{-5}I^2+1.79\times10^{-2}\ I+0.49$.

Arithmetic solution of the equation becomes an extremely laborious procedure primarily because of the complexity of the exponent a. Therefore, extensive use of the method for many locations over a long period of time requires the use of a tabular or graphical solution to the equation. Thornthwaite and Mather [4] have published such tables for only one location, Seabrook, N. J. Their publication includes a diagram for the graphical determination of unadjusted potential evapotranspiration in metric units. However, the day-length correction, depending on date and latitude, appears in a separate table. It was thought that a generalized and complete graphical solution in English units would be useful. Such a solution is presented here. It includes the adjustment for day length and provides a means whereby tables or graphs for use at any particular place can be readily prepared. In addition diagrams are provided for the conversion of weekly rates to daily rates of potential evapotranspiration and for a correction due to soil dryness.

Weekly rates of unadjusted potential evapotranspiration may be obtained from figure 1, using only the mean temperature for the period and the heat index (I) for the location concerned. The graph also provides the adjustment for day-length required to convert unadjusted potential evapotranspiration to adjusted potential evapotranspiration.

Figure 2 is a simple graph for converting weekly rates of adjusted potential evapotranspiration (PE) to daily rates. For certain purposes, daily rates may be desirable, although some evidence has been obtained which indicates that weekly rates of PE correlate better with measured evapotranspiration than do daily rates. A monthly rate can of course be obtained by simply multiplying the daily rate by the number of days in the month.

There is considerable evidence that during long dry periods the rate of evapotranspiration decreases as the soil dries. Thornthwaite and Mather [4] suggested one method of correcting for this effect. It is shown graphically in figure 3.

Listed below is the step-by-step procedure to be followed in the use of these graphs.

A. To obtain potential evapotranspiration, PE:

1. Using monthly normal or long-term mean temperatures (F.°) for the station or area concerned, obtain from table 1 the monthly heat index, i, corresponding to the normal temperature for each of the 12 months.
2. Add these 12 i-values to obtain I, the heat index.
3. Enter figure 1 at the bottom with the mean temperature (F.°) for the period concerned. (The length of the period is immaterial at this point; it may be of any length from 1 day to 1 month.)
4. Follow vertically up the appropriate mean temperature line until it intersects the horizontal heat index line equal to the heat index determined in step A-2 above. At this intersection determine unadjusted PE (inches) from the slanting lines. This is the potential moisture loss for a 7-day period, each day having a length of 12 hours.
5. Enter the bottom (or top) of the upper-right portion of figure 1 with the unadjusted PE value found in step A-4. This determines the appropriate vertical line to follow in this portion of the figure.
6. Enter the upper-left portion of figure 1 according to the north latitude of the station or area under consideration and proceed vertically downward to the intersection with the "month" line corresponding to the month under consideration. This intersection determines the appropriate horizontal line to follow into the upper-right portion of figure 1. (The climatological week numbers and the day-length correction are discussed in Appendix 1.)
7. At the intersection of the vertical line determined in step A-5 and the horizontal line determinéd in step A-6 read the final PE in inches per week from the curved lines.
8. If daily values are required, the daily mean temperature would be used in step A-3. The adjusted weekly PE rate obtained from step A-7 would then be converted to daily PE by dividing by 7 or by using figure 2.
9. If monthly PE is desired, monthly mean temperature would be used in step A-3 proceeding through step A-8 to obtain the mean daily rate for the month. This may be multiplied by the number of days in the month to obtain total PE for the month.

B. To correct PE to account for soil moisture depletion:

1. From actual measurement or from water balance tabulations, obtain the soil moisture content. This should be expressed as a percentage of the

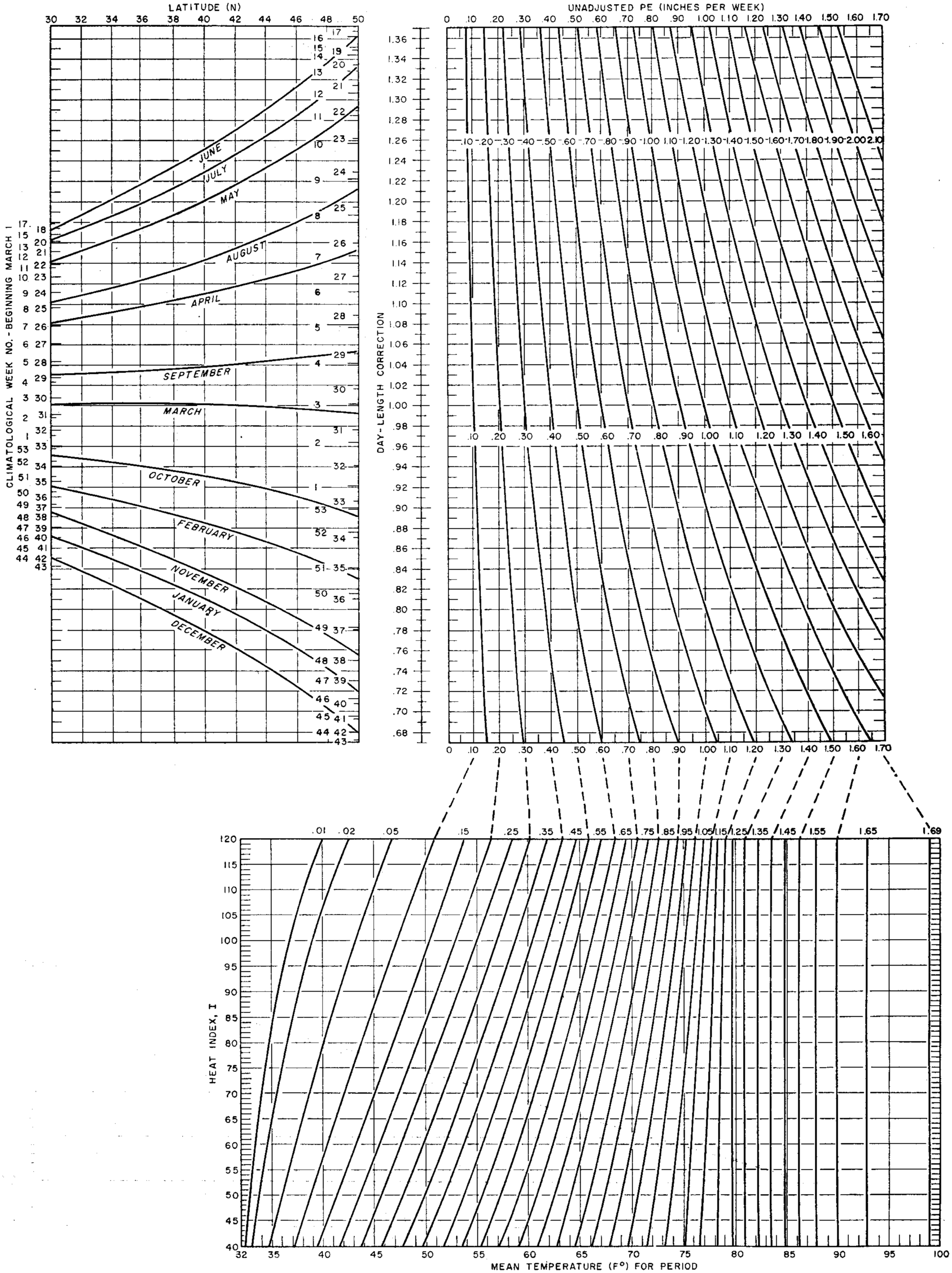

FIGURE 1.—Nomogram for computation of potential evapotranspiration (PE).

Table 1.—*Monthly values of i, according to monthly normal or long-term mean temperature (to tenths). To obtain I for use in figure 1, add the i-values obtained here for each of the 12 months.*

Monthly normal temperature (F°)	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
32	0	0	0	0	0.01	0.01	0.01	0.02	0.02	0.03
33	0.03	0.04	0.04	0.05	.06	.06	.07	.08	.09	.09
34	.10	.11	.12	.13	.14	.15	.15	.16	.17	.18
35	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28
36	.29	.31	.32	.33	.34	.35	.36	.37	.38	.39
37	.42	.43	.44	.45	.46	.47	.48	.49	.51	.53
38	.55	.56	.57	.58	.59	.61	.62	.63	.66	.68
39	.69	.70	.71	.72	.74	.75	.77	.80	.81	.82
40	.83	.85	.86	.88	.89	.91	.94	.95	.97	.98
41	1.00	1.01	1.03	1.04	1.06	1.09	1.10	1.12	1.14	1.16
42	1.17	1.19	1.20	1.22	1.25	1.27	1.29	1.30	1.32	1.33
43	1.35	1.37	1.39	1.42	1.43	1.45	1.47	1.49	1.50	1.52
44	1.54	1.56	1.59	1.61	1.63	1.64	1.66	1.68	1.70	1.72
45	1.74	1.77	1.79	1.81	1.83	1.85	1.87	1.89	1.91	1.93
46	1.96	1.98	2.00	2.02	2.04	2.06	2.08	2.10	2.12	2.15
47	2.17	2.19	2.21	2.23	2.25	2.27	2.29	2.31	2.35	2.37
48	2.39	2.41	2.44	2.46	2.48	2.50	2.52	2.56	2.58	2.60
49	2.62	2.64	2.66	2.69	2.71	2.73	2.76	2.79	2.81	2.83
50	2.86	2.88	2.90	2.92	2.94	2.99	3.01	3.03	3.05	3.08
51	3.10	3.12	3.14	3.17	3.20	3.23	3.25	3.27	3.30	3.32
52	3.34	3.36	3.39	3.42	3.45	3.48	3.50	3.53	3.55	3.58
53	3.60	3.62	3.67	3.69	3.72	3.74	3.76	3.78	3.81	3.83
54	3.86	3.90	3.93	3.96	3.98	4.00	4.02	4.05	4.07	4.10
55	4.14	4.17	4.20	4.22	4.25	4.27	4.30	4.32	4.35	4.39
56	4.42	4.45	4.47	4.50	4.52	4.55	4.57	4.60	4.64	4.67
57	4.70	4.72	4.75	4.79	4.81	4.83	4.86	4.90	4.93	4.96
58	4.98	5.01	5.04	5.07	5.09	5.12	5.16	5.19	5.22	5.25
59	5.28	5.30	5.33	5.35	5.39	5.43	5.46	5.49	5.52	5.55
60	5.57	5.60	5.63	5.66	5.70	5.73	5.76	5.79	5.82	5.84
61	5.87	5.90	5.93	5.98	6.01	6.04	6.07	6.10	6.12	6.15
62	6.17	6.21	6.26	6.29	6.32	6.35	6.38	6.41	6.44	6.47
63	6.50	6.54	6.57	6.60	6.63	6.66	6.69	6.72	6.75	6.78
64	6.84	6.81	6.90	6.92	6.95	6.98	7.01	7.04	7.08	7.12
65	7.15	7.19	7.22	7.25	7.28	7.31	7.34	7.38	7.42	7.45

Monthly normal temperature (F°)	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
66	7.49	7.52	7.55	7.58	7.61	7.64	7.68	7.72	7.75	7.79
67	7.82	7.85	7.88	7.91	7.94	7.98	8.02	8.06	8.10	8.13
68	8.16	8.19	8.22	8.25	8.29	8.33	8.37	8.41	8.44	8.47
69	8.50	8.53	8.56	8.60	8.64	8.68	8.72	8.75	8.78	8.81
70	8.85	8.88	8.91	8.96	9.00	9.04	9.07	9.10	9.13	9.17
71	9.20	9.23	9.28	9.32	9.36	9.39	9.42	9.45	9.49	9.52
72	9.56	9.60	9.64	9.68	9.71	9.75	9.78	9.82	9.85	9.88
73	9.93	9.97	10.01	10.04	10.08	10.11	10.15	10.18	10.22	10.26
74	10.30	10.35	10.38	10.41	10.44	10.48	10.52	10.56	10.60	10.64
75	10.68	10.71	10.75	10.78	10.82	10.85	10.89	10.94	10.98	11.02
76	11.05	11.09	11.12	11.16	11.19	11.23	11.28	11.32	11.37	11.40
77	11.44	11.47	11.50	11.54	11.58	11.62	11.66	11.70	11.74	11.78
78	11.82	11.86	11.90	11.94	11.98	12.02	12.06	12.10	12.14	12.18
79	12.22	12.26	12.30	12.34	12.37	12.41	12.45	12.49	12.53	12.57
80	12.61	12.65	12.69	12.73	12.77	12.81	12.85	12.89	12.93	12.97
81	13.01	13.05	13.09	13.13	13.17	13.21	13.25	13.29	13.33	13.37
82	13.41	13.45	13.49	13.53	13.57	13.61	13.65	13.69	13.73	13.77
83	13.81	13.85	13.89	13.93	13.97	14.01	14.05	14.09	14.14	14.19
84	14.24	14.28	14.32	14.36	14.40	14.44	14.48	14.52	14.57	14.61
85	14.65	14.69	14.73	14.77	14.81	14.85	14.90	14.95	14.99	15.03
86	15.07	15.11	15.15	15.19	15.23	15.28	15.33	15.38	15.42	15.46
87	15.50	15.54	15.58	15.62	15.66	15.70	15.75	15.79	15.84	15.88
88	15.92	15.96	16.01	16.06	16.10	16.15	16.19	16.23	16.27	16.31
89	16.35	16.39	16.44	16.49	16.54	16.58	16.62	16.66	16.70	16.74
90	16.78	16.83	16.88	16.93	16.97	17.01	17.05	17.09	17.13	17.18
91	17.23	17.28	17.33	17.37	17.41	17.45	17.49	17.53	17.58	17.63
92	17.68	17.73	17.77	17.81	17.85	17.89	17.93	17.98	18.03	18.08
93	18.13	18.17	18.22	18.26	18.30	18.34	18.39	18.44	18.49	18.54
94	18.58	18.62	18.66	18.70	18.75	18.80	18.85	18.90	18.95	18.99
95	19.03	19.07	19.11	19.16	19.21	19.26	19.31	19.36	19.41	19.45
96	19.49	19.53	19.58	19.63	19.68	19.73	19.78	19.82	19.86	19.90
97	19.95	20.00	20.05	20.10	20.15	20.20	20.24	20.28	20.32	20.37
98	20.42	20.47	20.52	20.57	20.62	20.66	20.70	20.75	20.80	20.85
99	20.90	20.95	21.00	21.04	21.09	21.13	21.17	21.21	21.25	21.30

total water-holding capacity in the root zone of the crop when soil moisture is at field capacity.

2. Enter figure 3 from the bottom with the daily PE value obtained from step A-8. Proceed upward along this line to the appropriate diagonal line representing the percentage value obtained in step B-1. Read the corrected evapotranspiration (E) along the diagonal scale at the left.

3. EXAMPLE

Find the evapotranspiration (E) for July 15 (climatological week No. 20) at a station located at 41° N. and having a heat index (I) of 52. Assume a daily mean temperature of 70° F. and a soil moisture content of 70 percent of field capacity. (Since the heat index (I) is given, steps A-1 and A-2 are not necessary.)

1. Proceed through step A-4, using the given mean temperature of 70° F. and I value of 52. This gives a weekly unadjusted PE rate of 0.92 inch.
2. Using the latitude of 41° N. and the date of July 15 proceed through steps A-5 to A-7 and determine the adjusted weekly PE rate of 1.13 inches.
3. To convert this weekly rate to a daily value proceed through step A-8 using figure 2. The final value is about 0.16 inch per day.
4. Finally, to correct this PE rate of 0.16 inch per day for the effect of soil moisture depletion, using the 70 percent of capacity as given, proceed through

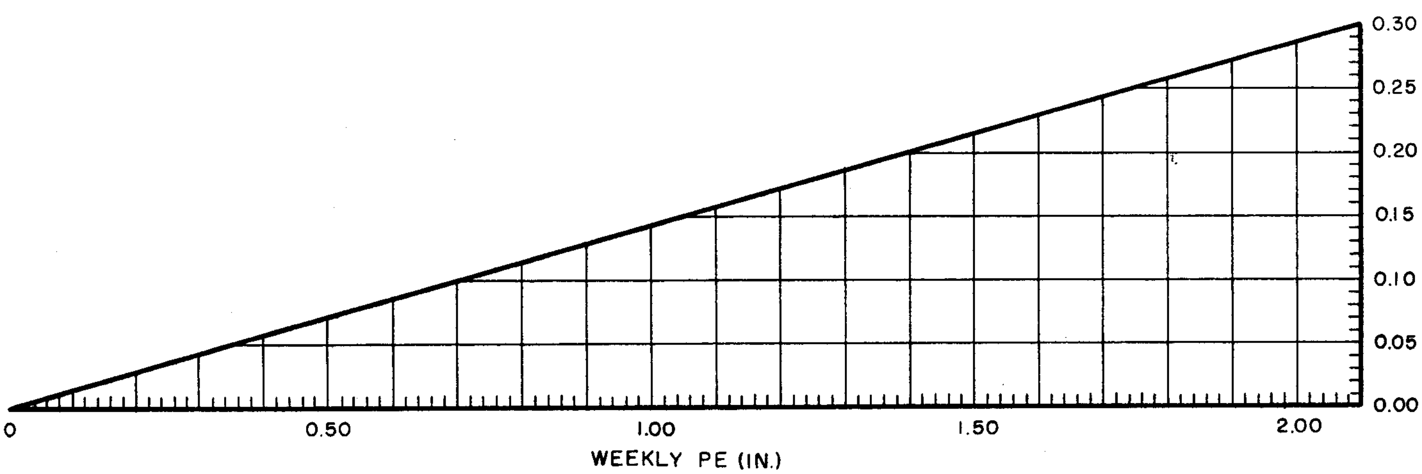

Figure 2.—Nomogram for conversion of weekly PE to daily PE.

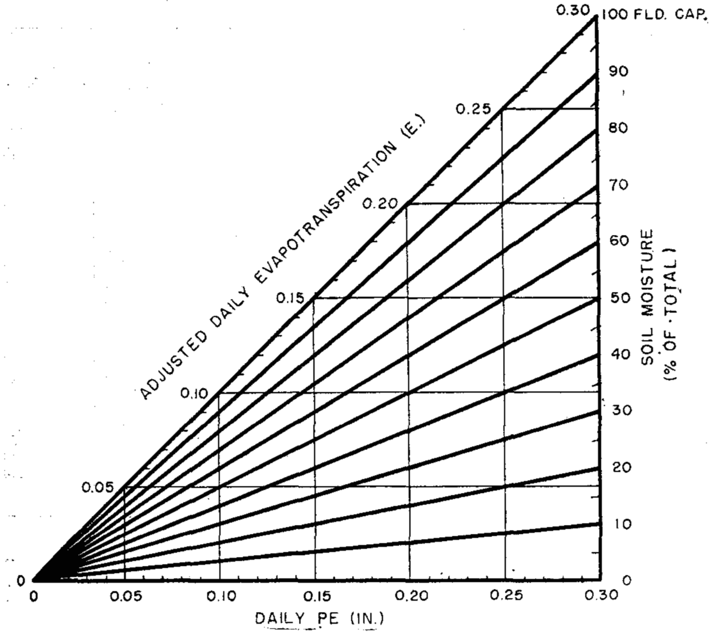

FIGURE 3.—Nomogram for adjustment of PE for soil moisture depletion.

step B-2 using figure 3. A daily rate of about 0.11 inch is determined, which is now an estimate of actual evapotranspiration rather than potential.

4. CONCLUSIONS

This graphical method is most useful for work involving a wide range of climatic conditions (i. e., a wide range of I values.) For computations covering a long period of time at a single station or a group of stations with nearly equal I and latitude values, it will usually save time to use tables or graphs which apply only to those particular stations. Such tables or graphs may easily be prepared using the diagrams presented here over a suitable range of temperature and time. For example, at a given place, latitude and heat index are constants and PE depends only on temperature and time of year; so a table of PE as a function of these two variables could be prepared from figure 1 without difficulty. (See Appendix 1.)

APPENDIX 1

The upper-left portion of figure 1 concerns the correction for latitude and time of year. The unadjusted PE obtained in the lower portion of figure 1 must be multiplied by this day-length correction in order to obtain adjusted rate of weekly PE. The curved lines labeled by months represent mid-month, and for most purposes interpolation between these lines according to date will suffice. However, for a more accurate determination one may use climatological week number (March 1–7=week No. 1, 8–14=No. 2, etc.) rather than month. These lines are not reproduced, but the week numbers are shown for the middle day of the week at 30° and 50° N. latitude. With an appropriate drafting curve one can construct the

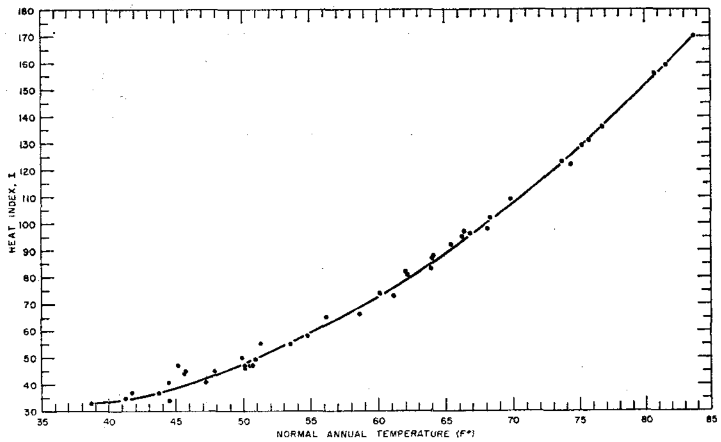

FIGURE 4.—Relation of heat index, I, to normal annual temperature.

lines for week number. For example, the curve for week No. 10 (May 3–9) would have approximately the same shape as the May line, whereas week No. 24 (Aug. 16–22) would approximate the August curvature.

It is not necessary that the numerical value of the day-length correction be determined; the adjustment is accomplished graphically when the appropriate horizontal line of day-length correction is followed to the right until it intersects the appropriate vertical line of unadjusted PE and the final PE is read from the curved lines. However, in order to facilitate the work of those who may wish to prepare tables for a particular station, the scale for day-length correction has been included.

The preparation of tables for any given location is a simple matter. A table of unadjusted PE as a function of temperature can be prepared from the lower portion of figure 1 along the appropriate heat index line. The day-length correction table can be prepared from the upper-left portion of figure 1 by tabulating the value of the correction for each month (or week) along the appropriate latitude line. The final PE for any particular week is then the product of the appropriate two values from the tables. Another convenient device for use at a particular station is a set of tables or graphs of final PE as a function of temperature for each of the 12 months. If these tables are made for monthly PE, an adjustment for month length must be made as indicated in step A-9.

Figure 1 as drawn applies to the Northern Hemisphere. It could be adapted for use in the Southern Hemisphere by the appropriate relabeling of the month lines in the upper-left portion.

APPENDIX 2

Inasmuch as monthly heat index, i, is a function of temperature only [3]

$$i=(T/5)^{1.514},$$

it seemed reasonable to investigate the relationship between heat index, I, as determined from table 1, and normal annual temperature. This was done for the 43 stations shown in the accompanying table 2. The only criterion for the selection of these stations was that a

TABLE 2.—*Normal annual temperature and heat index value, I. (Arranged in order of increasing temperature.)*

Station	Normal annual temperature (°F.)	Heat index, *I*, from table 1	Station	Normal annual temperature (°F.)	Heat index *I*, from table 1
Devils Lake, N. Dak	38.7	33	Raleigh, N. C	61.1	73
Williston, N. Dak	41.3	35	Fort Smith, Ark	62.0	82
Bismarck, N. Dak	41.7	37	Atlanta, Ga	62.2	81
Havre, Mont	43.7	37	Los Angeles, Calif	63.9	83
Portland, Maine	44.5	34	Augusta, Ga	64.0	87
Rochester, Minn	44.5	41	Abilene, Tex	64.1	88
Minneapolis, Minn	45.6	44	Montgomery, Ala	65.4	92
Huron, S. Dak	45.7	45	San Angelo, Tex	66.2	95
Billings, Mont	47.2	41	Shreveport, La	66.4	97
Valentine, Nebr	47.9	45	Savannah, Ga	66.8	96
Goodland, Kans	49.9	50	Mobile, Ala	68.1	98
Scranton, Pa	50.1	46	Lake Charles, La	68.3	102
Hartford, Conn	50.1	47	Del Rio, Tex	69.8	109
Bridgeport, Conn	50.5	47	Brownsville, Tex	73.6	123
Boston, Mass	50.7	47	Laredo, Tex	74.3	122
Allentown, Pa	50.9	49	Honolulu, Hawaii	75.2	129
Burlington, Iowa	51.3	55	Miami, Fla	75.7	131
Trenton, N. J	53.5	55	Ponce Santa Isobel, West Indies	76.7	136
Baltimore, Md	54.7	58	Swan Island, West Indies	80.7	156
Kansas City, Mo	56.1	65	Yap, Caroline Islands	81.6	159
Richmond, Va	58.6	66	Canton, Phoenix Islands	83.7	170
Nashville, Tenn	60.1	74			

wide and uninterrupted range of normal annual temperatures should be represented. As a consequence a number of climatic types are represented but not equally so.

Figure 4 shows heat index plotted as a function of normal annual temperature for these 43 points. The curve was drawn by eye. The relationship is surprisingly good and suggests that heat index, *I*, can be quickly estimated from normal annual temperature with a maximum error of $\pm$ 4 or 5. This amount of error in the heat index would introduce an error in PE of no more than .03 or .04 inch per week, which is very likely well within the limits of accuracy of the entire empirical procedure. Of course, this result suggests that the heat index scale in figure 1 could be replaced by a scale of normal annual temperature without significant loss of accuracy.

REFERENCES

1. W. R. van Wijk and D. A. de Vries, "Evapotranspiration," *Netherlands Journal of Agricultural Science*, vol. 2, pp. 105–119, 1954.
2. H. L. Penman, "Evaporation: An Introductory Survey," *Netherlands Journal of Agricultural Science*, vol. 4, pp. 9–29, 1956.
3. C. W. Thornthwaite, "An Approach toward a Rational Classification of Climate," *Geographical Review*, vol. 38, pp. 55–94, 1948.
4. C. W. Thornthwaite and J. R. Mather, "The Water Balance," *Publications in Climatology*, vol. 8, No. 1, Drexel Institute of Technology, Laboratory of Climatology, Centerton, N. J., 1955.
5. H. F. Blaney and W. D. Criddle, *Determining Water Requirements in Irrigated Areas from Climatological and Irrigation Data*, U. S. Department of Agriculture, Soil Conservation Service, 1950.
6. C. H. M. van Bavel, *Estimating Soil Moisture Conditions and Time for Irrigation with the Evapotranspiration Method*, U. S. Department of Agriculture, Agricultural Research Service, 1956.
7. G. W. Robertson, *Latent Evaporation: Its Concept, Measurement, and Application*, Experimental Farms Service, Department of Agriculture, Ottawa, Canada, 1954.